

拉格朗日插值公式

例 1：已知 $f(x)$ 為一 3 次多項式，且 $f(0) = -16, f(1) = 2, f(2) = 3, f(3) = 1$ ，試求 $f(x)$ ？

解一：設 $f(x) = ax^3 + bx^2 + cx + d$ ，將 $f(0) = -16, f(1) = 2, f(2) = 3, f(3) = 1$ 代入，可得四個聯立方程組，四個聯立式解 4 個未知數 a, b, c, d ，故可解得 $f(x)$

$$\begin{aligned} \text{解二：設 } f(x) &= (-16) \frac{(x-1)(x-2)(x-3)}{(0-1)(0-2)(0-3)} \\ &\quad + (2) \frac{(x-0)(x-2)(x-3)}{(1-0)(1-2)(1-3)} \\ &\quad + (3) \frac{(x-0)(x-1)(x-3)}{(2-0)(2-1)(2-3)} \\ &\quad + (1) \frac{(x-0)(x-1)(x-2)}{(3-0)(3-1)(3-2)} \\ &= \frac{8(x-1)(x-2)(x-3)}{3} - x(x-2)(x-3) - \frac{3x(x-1)(x-3)}{2} + \frac{x(x-1)(x-2)}{6} \\ &= \frac{1}{3}x^3 - \frac{11}{2}x^2 + \frac{115}{6}x - 16 \end{aligned}$$

練習 1：已知 $f(x)$ 為一 2 次多項式，且 $f(1) = 2, f(2) = 3, f(3) = 1$ ，試求 $f(x)$ ？

練習 2：已知 $f(x)$ 為一 3 次多項式，且 $f(0) = -1, f(1) = 1, f(2) = 2, f(3) = 3$ ，試求 $f(x)$ ？

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n 次多項式 $f(x)$ ，若 $f(x_0) = y_0, f(x_1) = y_1, \dots, f(x_n) = y_n$ ，則 $f(x) =$

$$\begin{aligned} &y_0 \times \frac{(x-x_1)(x-x_2)\dots(x-x_n)}{(x_0-x_1)(x_0-x_2)\dots(x_0-x_n)} + y_1 \times \frac{(x-x_0)(x-x_2)\dots(x-x_n)}{(x_1-x_0)(x_1-x_2)\dots(x_1-x_n)} + \\ &y_2 \times \frac{(x-x_0)(x-x_1)\dots(x-x_n)}{(x_2-x_0)(x_2-x_1)\dots(x_2-x_n)} + \dots + y_n \times \frac{(x-x_1)(x-x_2)\dots(x-x_{n-1})}{(x_n-x_1)(x_n-x_2)\dots(x_n-x_{n-1})} \end{aligned}$$

證明：將 x_0 代入，則 $f(x_0) = y_0$ 正確

將 x_1 代入，則 $f(x_1) = y_1$ 正確

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將 x_n 代入，則 $f(x_n) = y_n$ 正確

故 $f(x) =$

$$\begin{aligned} &y_0 \times \frac{(x-x_1)(x-x_2)\dots(x-x_n)}{(x_0-x_1)(x_0-x_2)\dots(x_0-x_n)} + y_1 \times \frac{(x-x_0)(x-x_2)\dots(x-x_n)}{(x_1-x_0)(x_1-x_2)\dots(x_1-x_n)} + \\ &y_2 \times \frac{(x-x_0)(x-x_1)\dots(x-x_n)}{(x_2-x_0)(x_2-x_1)\dots(x_2-x_n)} + \dots + y_n \times \frac{(x-x_1)(x-x_2)\dots(x-x_{n-1})}{(x_n-x_1)(x_n-x_2)\dots(x_n-x_{n-1})} \end{aligned}$$

滿足題目所給各項條件 $f(x_0) = y_0, f(x_1) = y_1, \dots, f(x_n) = y_n$