

恆等多項式

$$1. (x+y)^n = x^n + nx^{n-1}y + \frac{n(n-1)}{2!}x^{n-2}y^2 + \dots + \frac{n!}{k!(n-k)!}x^{n-k}y^k + \dots + nxy^{n-1} + y^n$$

$$= \sum_{k=0}^n \frac{n!}{k!(n-k)!} x^{n-k} y^k$$

$$= \sum_{k=0}^n C_k^n x^{n-k} y^k$$

$$2. (x-y)^n = \sum_{k=0}^n \frac{n!}{k!(n-k)!} x^{n-k} (-y)^k$$

$$= \sum_{k=0}^n \frac{n!}{k!(n-k)!} (-1)^k x^{n-k} y^k$$

$$= \sum_{k=0}^n C_k^n (-1)^k x^{n-k} y^k$$

$$3. x^n - y^n = (x-y)(x^{n-1} + x^{n-2}y + x^{n-3}y^2 + \dots + xy^{n-2} + y^{n-1})$$

$$4. x^n + y^n = (x+y)(x^{n-1} - x^{n-2}y + x^{n-3}y^2 - \dots - xy^{n-2} + y^{n-1}), n \text{ 爲奇數}$$

$$5. x^n + y^n = (x+y)(x^{n-1} - x^{n-2}y + x^{n-3}y^2 - \dots + xy^{n-2} - y^{n-1}), n \text{ 爲偶數}$$

$$6. (x_1 + x_2 + x_3 + \dots + x_n)^2 = (x_1^2 + x_2^2 + x_3^2 + \dots + x_n^2) + 2(\text{兩兩乘積和})$$

練習 1：試展開 $(x+1)^6$

練習 2：試展開 $(x-1)^7$

練習 1：試求 1,3,5,7,9,11,13,15,17,19,21,23,25,27,29 等數的兩兩乘積和