

$$\begin{aligned}
1. \quad & 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6} + \frac{1}{7} + \frac{1}{8} + \frac{1}{9} + \frac{1}{10} + \frac{1}{11} + \frac{1}{12} + \frac{1}{13} + \frac{1}{14} + \frac{1}{15} + \frac{1}{16} + \frac{1}{17} + \frac{1}{18} + \dots \\
& > 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{4} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{16} + \frac{1}{16} + \frac{1}{16} + \frac{1}{16} + \frac{1}{16} + \frac{1}{16} + \frac{1}{16} + \frac{1}{16} + \frac{1}{32} + \frac{1}{32} + \dots \\
& = 1 + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \dots \\
& = 1 + 1 + 1 + \dots \\
& \approx \infty \quad \text{故 } \sum_{n=1}^{\infty} \frac{1}{n} \text{ 發散}
\end{aligned}$$

$$\begin{aligned}
2. \quad & 1 + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \frac{1}{5^2} + \frac{1}{6^2} + \frac{1}{7^2} + \frac{1}{8^2} + \frac{1}{9^2} + \frac{1}{10^2} + \frac{1}{11^2} + \frac{1}{12^2} + \frac{1}{13^2} + \frac{1}{14^2} + \frac{1}{15^2} + \dots \\
& \quad \frac{1}{16^2} + \frac{1}{17^2} + \frac{1}{18^2} + \dots \\
& < 1 + \frac{1}{2^2} + \frac{1}{2^2} + \frac{1}{4^2} + \frac{1}{4^2} + \frac{1}{4^2} + \frac{1}{4^2} + \frac{1}{8^2} + \frac{1}{8^2} + \frac{1}{8^2} + \frac{1}{8^2} + \frac{1}{8^2} + \frac{1}{8^2} + \frac{1}{8^2} + \frac{1}{8^2} + \frac{1}{8^2} + \dots \\
& = 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \dots \\
& = \frac{1}{1 - \frac{1}{2}}
\end{aligned}$$

$$\approx 2 \quad \text{故 } \sum_{n=1}^{\infty} \frac{1}{n^2} \text{ 收斂}$$

$$\begin{aligned}
3. \quad & \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \dots + \frac{1}{(2n+1)^2} \\
& < \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{4^2} + \dots + \frac{1}{(2n)^2} \\
& = \frac{1}{1^2} + \frac{1}{2^2} \left( \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \frac{1}{5^2} + \dots + \frac{1}{n^2} \right) \\
& < 1 + \frac{1}{4} \left( 2 - \frac{1}{n} \right) \\
& = \frac{3}{2} - \frac{1}{4n}
\end{aligned}$$